M Math Analysis of several variables Back paper examination .

Answer all the 10 questions. Each question is worth 10 points.

If you are using any result proved in the class, you need to state it correctly. If the answer is an immediate consequence of a result quoted, then that result also needs to be proved.

1. Let $T : R^n \to R^m$ be a linear map. Show that there exists a $k > 0$ such that $\|T(x)\| \leq k\|x\|$ for all $x \in R^n$.
2. Let $f, g : R^3 \to R$ be differentiable functions. Give a detailed proof to show that fg is a differentiable function.
3. Give an example of a differentiable function $F : R^2 \to R$, that has no local maximum and no local minimum at $(0, 0)$.
4. Give an example with full details of a function $f : R^2 \to R$ for which $D_{12}f$ and $D_{21}f$ exists but are not equal.
5. Let $f : \{(x, y) \in R^2 : x + y < 1\} \to R$ be a convex function. Show that $F : [0, 1] \to R$ defined by $F(t) = f(\frac{1}{2}(1-t), \frac{t}{2})$ for $t \in [0, 1]$ is a convex function.
6. Let $f : (0, 1) \times (0, 1) \to R$ be a differentiable function. Suppose $f'((0, 1) \times (0, 1)) \subset B(0, \frac{1}{2})$. Show that f is uniformly continuous.
7. Let $f : (0, 1) \to \{x \in R^n : x_1^2 + ... + x_n^2 = 1\}$ be a differentiable mapping. Show that the dot product $f.f' = 0$
8. Let $Q^3 = \{(x, y, z) \in [0, 1] \times [0, 1] \times [0, 1] : x + y + z \leq 1\}$. Show that $\int_{Q^3} e^{x+2y+3z} = \frac{(e-1)^3}{6}$.
9. Let $f : [0, 1] \times [0, 1] \to R$ be a continuous function. Show that $\int_{[0,1]\times[0,1]} f dx dy = \int_0^1 (\int_0^1 f(x, y) dx) dy$.
10. Let ω, σ, λ be continuously differentiable 2-forms and a 3-form respectively. Show with full details that $\lambda \wedge (\omega + \sigma) = \lambda \wedge \omega + \lambda \wedge \sigma$.